Mathematics competition 4.2.2011 in Finnish comprehensive school Final Round

SECTION 1   Solution time 30 min   Maximum score 20

**Students are not allowed to use calculator in this section.**
Present your working, picture or any short explanations.
You can get up to two points for tasks 1-4 and up to three points for tasks 5-8.

1. The lengths of the sides of a rectangle are 1 and 2 (unit length).
   Divide it into parts so that they can form a square. Draw your solution.

2. The values of a pentagon's angles are consecutive integers.
   How large are the angles?

3. The figure can be put together into a cube. What is the biggest possible
   product of numbers around one vertex of the cube?
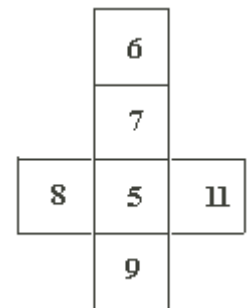
4. Integers from 1 to 20 are written in succession without spaces:
   1234567891011121314151617181920.
   Strike out 21 digits; so that the remaining digits form the greatest possible
   number, when you write them without changing the order. What number
   did you get?

5. Olli wrote a three-digit number, and then he calculated the sum of the digits, and finally the
   sum of its digits. When digits are replaced by corresponding shapes , all three numbers can
   be written like this:

   $\diamond \, \bigcirc \, \diamond$ $\bigcirc \, \triangle$ $\bigcirc$
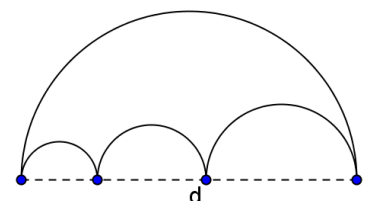
   What numbers did Olli write?

6. The radius of the circle is 10 (unit length). It is divided into five parts of
   equal area. One part is a square (see picture). Calculate the length of the
   square side. Round the answer to the nearest integer.
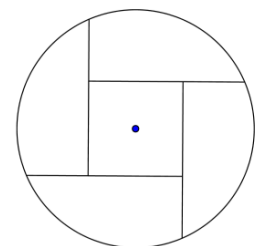
7. What is the value of **a : c,** if **a : b = 3 : 4** and **a : (b + c) = 2 : 5**?

8. How long is the perimeter of the figure, formed by the semi-circles?
   (d =the diameter of the largest semi-circle, marked with a dotted line)

SECTION 2   Solution time 45 min   Maximum score 20

Domino tile is a rectangle composed of two squares. Dots on the tile are called pips. The most common *domino set* has from 0 to 6 pips in each square. In this case, the entire set contains tiles with pips 00, 01, 11, 02, 12, 22, 03, ..., 56, 66, in other words, the smallest number of pips is 0, the largest number of pips is 6, and the set does not contain two identical tiles. There are also domino sets with number of pips from 0 to 9.

   The following *placement rule* is used in the tasks below unless stated otherwise. The tiles are placed in a row one after another in a single direction so that each tile must have the same number of pips on its first square as the previous tile has on its second square.

   For instance, the line presented on the picture is constructed from a set with the number of pips from 0 to 6. The first tile located at the top left corner is 04. The whole tile line can be described as 04-41-15-52-26-63. In that case the *line length* is six tiles.
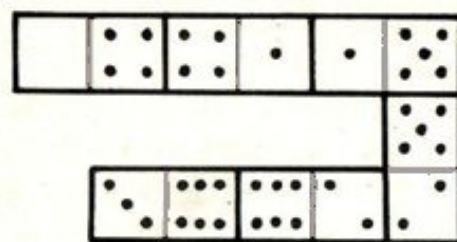
   In the presented line we have used an *additional rule* that the sum of tile's pips increases by one.

1. There is a domino set, which has pips from 0 to 6.
    The placement rule is valid. Describe each of the lines.

    a) How long line can you get, if you start from tile 23, and the sum of pips must increase by two and continue as long as possible?

    b) How long line you can get, if you start from any tile and the sum of tile's pips increases by three and form the longest possible line?

    c) Make the longest possible line with increasing sum of tile's pips.
        You can choose both the first tile and the step of increase.

                                                                      (3 points)

2. How long line can you make if you use a domino set with pips from 0 to 9 and you can choose the first tile and the step of increase? Describe the line.
                                                                      (2 points)

3. How many tiles will be in the set if the number of pips is
    a) from 0 to 9
    b) from 0 to n?   *(4 points)*

   TURN OVER!

For the next tasks use domino set with pips from 0 to 6. The placement rule is not valid.

4. The side faces of the parallelepiped on the picture consist of one domino tile each, the top and bottom faces each consist of two tiles. The sum of pips on every face is the same.
   a) How big can be the sum of pips on a face of the parallelepiped?
   b) How many different combinations of tiles for the top face are possible?
      What are they?
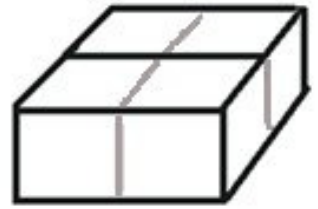
(5 points)

5. Four tiles are chosen so that they make a square that has the same sum of pips on each side. The sample square presented on the picture has the sum of pips on every side equal to 11.
   Make as many as possible squares like that from a single domino set. The sums of pips on sides of those squares can be different. The only requirement is that within one square the sums must be the same. The presented square is not necessary a part of a solution.
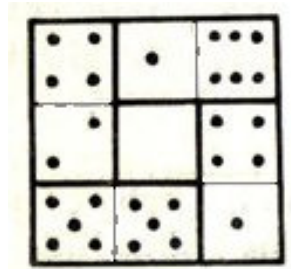
(6 points)

## SECTION 3   Solution time 60 min   Maximum score 30

Show your work or present an explanation. Each task is 6 points.

1. Marta, Ville and Teppo each have two professions, which others do not have. These professions are writer, architect, teacher, doctor, lawyer and artist. Professions mentioned in the same tip belong to different people.

    1) The writer and the teacher went skiing with Marta.
    2) The doctor asked the artist to paint his portrait.
    3) The doctor and the teacher had a joint meeting.
    4) The artist is a relative of the architect.
    5) Teppo won Ville and the artist in chess.
    6) Ville and the writer live in the neighbourhood.

    Which professions does each person have?

2. Three towers of cellular phone network are located so that the distance between any two of them is 6.00 km. The signal of each tower covers a circular area with a radius of about 3.5 km. The area covered by all three towers is a point. Calculate the size of the area covered by two towers.
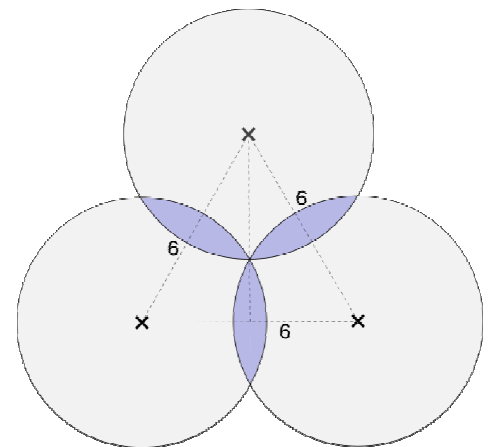
3. Calculate the value of the following expression
    $$(a + b)(a^2 + b^2)(a^4 + b^4) \dots (a^{2^n} + b^{2^n}) \, ,$$
    if $a = 10$, $b = 1$ and $n = 5$.

4. Vertices D and E of triangle DEF are fixed, and vertex F moves along the sides of triangle ABC counterclockwise starting from point A. Draw the function graph for the area of triangle DEF depending on the distance covered by point F. Choose a suitable coordinate system.
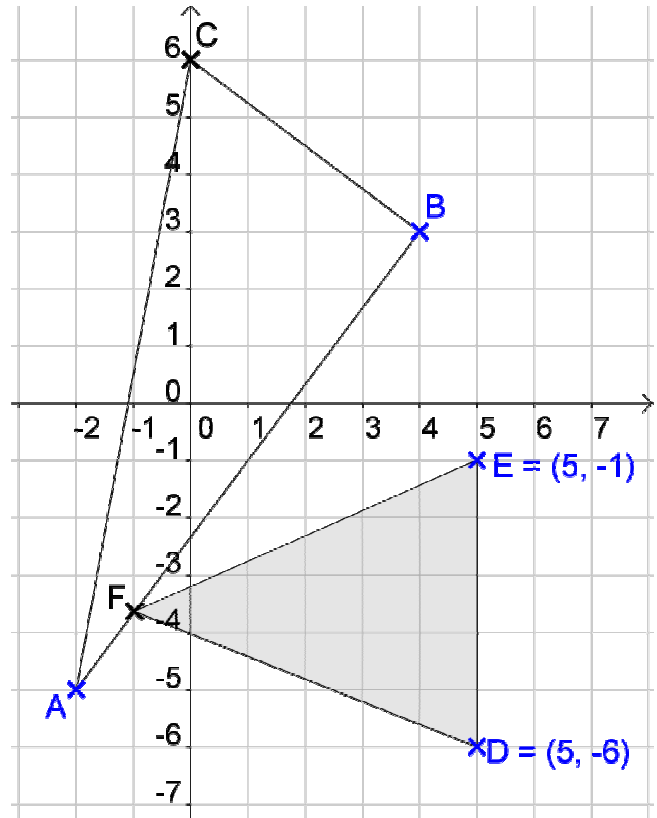
5. A palindrome is a positive number that is the same when read forwards or backwards. For example, the numbers 5, 232 and 4428244 are palindromes.

   a) Determine the number of palindromes with seven digits.

   b) If the palindromes in part (a) are written in increasing order, determine the 2125th palindrome in the list.