VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

2016 VCE Mathematical Methods 1 examination report

General comments

The 2016 Mathematical Methods 1 examination was the first examination for the revised study, and the format and structure of the examination were similar to those of previous years. Completion of past examinations (available on the VCAA website) will develop examination technique.

The majority of students made reasonable attempts at all questions or parts thereof. This was particularly evident for Question 5. Students who ensured that their responses clearly and neatly communicated their thought processes, as well as demonstrated depth of understanding of the concepts or formulations utilised, were rewarded. Students should take care with the setting out of their solutions and working to avoid making many complicated arithmetic or algebraic manipulations within a single line. Derivatives by rule, sketch graphs and determining a rule for a composite or inverse function were well handled.

Students are urged to read questions carefully and ensure that they answer the specific question asked. The terms 'label' (Question 3a.), 'show that' (Question 5aiii.), 'show by' (Question 8a.) and 'hence' (Question 8bii.) required an explicit response. Such questions needed to be read more carefully. The wording of Questions 4c., 7a. and 7b. directed students to express their final answer in a specific format.

Some good reasoning was marred by poor use of notation. Students are reminded that mathematics is a precise language. The interval $[0,\infty)$ is not the same interval as $(0,\infty)$. This was a common error in Questions 5aii. and 5bii., where domains and ranges were required. In the same vein, a logarithmic expression requires a base. The expressions loge(x) and loge$^{(x)}$ are not the equivalents of $\log_e(x)$. Poor usage of brackets, particularly evident in Questions 1a., 5aiii. and 8a., often led to errors that could have been avoided. Students are encouraged to practise their arithmetic skills, particularly fractions involving sums of reciprocals in the denominator (Question 6).

VICTORIA State Government

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1a.

Marks	0	1	2	Average
%	11	33	56	1.5

$$\frac{dy}{dx} = \frac{-\sin(x)(x^2+2) - 2x\cos(x)}{(x^2+2)^2}$$

Most students were able to confidently apply the quotient rule. However, many students did not obtain full marks due to errors caused by, for example, a denominator of x^4+4 as the supposed expansion of $\left(x^2+2\right)^2$. Students should very carefully consider the placement and usage of brackets. For example, the expression $x^2+2\times-\sin(x)$ is not equivalent to $(x^2+2)\times-\sin(x)$.

Question 1b.

Marks	0	1	2	Average
%	13	18	69	1.6

$$f'(x) = 2xe^{5x} + 5x^2e^{5x}$$

$$f'(1) = 7e^5$$

This question was well answered. Most students correctly identified the product rule but did not evaluate (as instructed) or their answers were incomplete. An incorrect combination of the product and chain rule resulted in an answer of $10xe^{5x}$ as a common error.

Question 2a.

Marks	0	1	Average
%	31	69	0.7

$$f'(x) = -\frac{1}{\sqrt{1-2x}}$$

Most students utilised the chain rule on an expression involving a fractional exponent. However, many students then missed the negative sign in the final answer, forgetting that the derivative of $(1-2x)$ is -2.

Question 2b.

Marks	0	1	2	Average
%	42	39	20	0.8

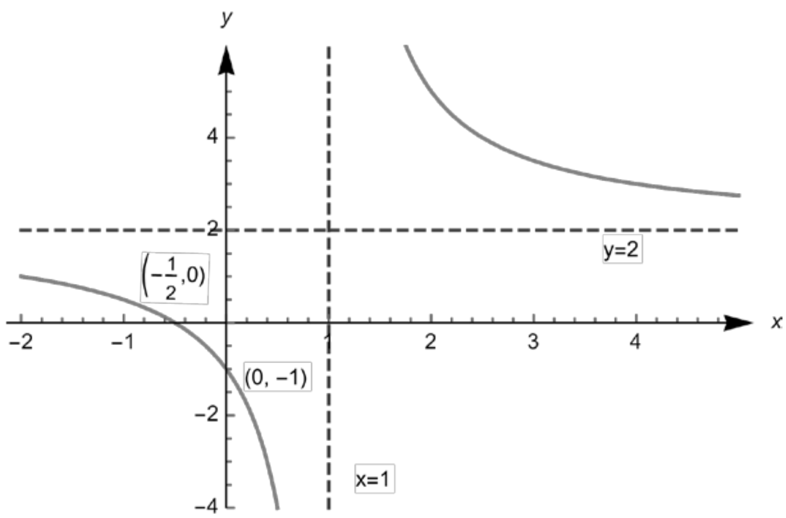
$$\tan\theta = f'(-1) = -\frac{1}{\sqrt{3}}$$

$$\theta = \frac{5\pi}{6}^{c} \ \ or \ \ 150^{\circ}$$

This question was not answered well. Many students who knew the connection between $\tan(\theta)$ and $f'(-1)$ had difficulty in finding the required angle. Students should know the exact values of circular functions in all quadrants. Many students incorrectly assumed that $\text{gradient} = f(-1)$ or wasted time finding the equation of the tangent.

Question 3a.

Marks	0	1	2	3	Average
%	9	10	24	57	2.3



Some well-constructed graphs were presented by students. The highest-scoring graphs were clearly labelled with the correct points/equations, as specified by the question, and with care taken in showing the asymptotic behaviour nature of the curve as it approached an asymptote. Using a dashed line to represent an asymptote indicated that the curve was distinct from its asymptote.

Question 3b.

Marks	0	1	2	Average
%	25	29	46	1.2

$$\text{Area } = \int_2^4 2 + \frac{3}{x-1} dx$$

$$= \left[2x + 3\log_e(x-1)\right]_2^4$$

$$= 4 + 3\log_e(3) \text{ or alternatively } 4 + \log_e(27)$$

Students were able to identify the required integral; however, many then erred in the evaluation of the terminals. A common error occurring as a result of incorrectly applying logarithmic laws was a final answer of $4+\log_e(9)$. Many students were unable to recognise that the antiderivative of the reciprocal of $(x-1)$ involved a logarithmic function. The '*dx*' was often omitted.

Question 4a.

Marks	0	1	Average
%	42	58	0.6

$X \sim \text{Bi}(4, \tfrac{1}{3})$

$$\Pr(X = 0) = \frac{16}{81}$$

Incorrect responses overlooked the stipulation 'four times each day', thus not identifying a binomial distribution. Some students considered untagged sheep rather than tagged sheep.

Question 4b.

Marks	0	1	Average
%	42	58	0.6

$$Pr(X \geq 1) = 1 - Pr(X = 0) = \frac{65}{81}$$

Students identified that the answer to this part of the question was simply the complement of their previous answer. However, some students wasted time in finding the sum of four probabilities, and others made arithmetic errors.

Question 4c.

Marks	0	1	Average
%	39	61	0.6

$Y \sim Bi\left(6, \frac{16}{81}\right)$

$\Pr(Y = 0) = \left(\frac{16}{81}\right)^6$ or $\left(\frac{2}{3}\right)^{24}$

The majority of students made the correct connection to part a. and the exponent 6. The most common incorrect answer was $\left(\frac{2}{3}\right)^6$.

Question 5ai.

Marks	0	1	Average
%	5	95	1

$h(x) = \log_e(x^2+1)$

This question was very well answered. The few errors tended to be the result of poor notation, for example $\log_e x^2 + 1$, rather than a lack of understanding in determining the rule of this composite function.

Question 5aii.

Marks	0	1	2	Average
%	55	30	15	0.6

Domain: R

Range: $[0, \infty)$ *or* $R^+ \cup \{0\}$

A small proportion of students gained full marks for this question. While poor notation was a contributing factor, students appeared to experience difficulty in determining the range of the composite function, *h*. A quick sketch over the given domain would have been helpful. Students are reminded that the domain and range of a function are key aspects of a function.

Question 5aiii.

Marks	0	1	2	Average
%	27	42	31	1.1

$=h(x)+h(-x)$

$$=\log_e\left(x^2+1\right)+\log_e\left((-x)^2+1\right)$$

$$=\log_e\left(x^2+1\right)+\log_e\left(x^2+1\right)$$

$$=2\log_e\left(x^2+1\right)$$

$=f\left(\left(g(x)^2\right)\right)$

$$=\log_e\left(\left(x^2+1\right)^2\right)$$

$$=2\log_e\left(x^2+1\right)$$

Hence $h(x)+h(-x)=f\left(\left(g(x)^2\right)\right)$

Many students were unsure of how to present their working. In the sample working above, both sides were operated on separately to arrive at the same expression and the conclusion that one side was in fact equivalent to the other. Poor notation was again evident, in particular $\log_e(-x^2+1)\neq\log_e((-x)^2+1)$.

Question 5aiv.

Marks	0	1	2	Average
%	45	28	27	0.8

$h'(x)=\dfrac{2x}{x^2+1}=0$ when $2x=0$ *i.e.* $x=0$

Minimum stationary point at $(0,0)$

Methods to identify the nature of the stationary point included:

- use symmetry of g and log as an increasing function
- use of range found in 5aii.
- use of graph
- a sign table.

Most students could equate the correct derivative to 0. However, many then were unable to solve the equation, forgetting that a fraction is zero when its numerator is zero. Of those who managed a solution, some overlooked the second part of the question or assumed that the stationary point was a point of inflection.

Question 5bi.

Marks	0	1	2	Average
%	18	65	16	1

To obtain an expression for the rule of the given function:

Let $x = \log_e(y^2 + 1)$

$e^x = y^2 + 1$

$y^2 = e^x - 1$

$y = \pm\sqrt{e^x - 1}$

applying domain and range considerations:

$y = -\sqrt{e^x - 1}$

Hence $k^{-1}(x) = -\sqrt{e^x - 1}$

Students appeared quite adept at the mechanics of determining the rule for the inverse: swap x and y then rearrange. However, few students took care to determine the range of the inverse function and select for the negative root of their expression.

Question 5bii.

Marks	0	1	2	Average
%	36	40	24	0.9

Domain: $[0, \infty)$ *or* $R^+ \cup \{0\}$

Range: $(-\infty, 0]$ *or* $R^- \cup \{0\}$

This question was not answered well. Most students utilised the fact that $Range_{k^{-1}} = Domain_k$ but found stating the domain of the inverse function more difficult. Again, poor notation was evident.

Question 6a.

Marks	0	1	2	Average
%	37	31	32	1

$f\left(-\frac{\pi}{3}\right) = 2\sin\left(-\frac{2\pi}{3}\right) - 1 = -\sqrt{3} - 1$

$f\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) - 1 = \sqrt{3} - 1$

$$\text{Average rate of change } = \frac{(\sqrt{3}-1)-(-\sqrt{3}-1)}{\frac{\pi}{6}-\left(-\frac{\pi}{3}\right)}$$

$$= \frac{4\sqrt{3}}{\pi}$$

Most students used the correct gradient rule but erred when evaluating, particularly $f\left(-\frac{\pi}{3}\right)$ or in dealing with fractions in the denominator. A few students confused average rate of change with average value, and some incorrectly found the average of derivatives.

Question 6b.

Marks	0	1	2	3	Average
%	31	23	30	16	1.3

$$\text{Average value} = \frac{1}{\frac{\pi}{6}-\left(-\frac{\pi}{3}\right)} \int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} 2\sin(2x) - 1\, dx$$

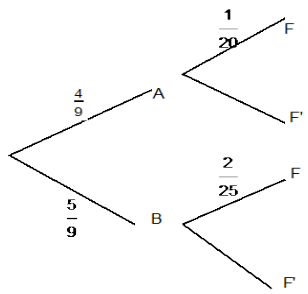
$$= \frac{2}{\pi}\left[-\cos(2x) - x\right]_{-\frac{\pi}{3}}^{\frac{\pi}{6}}$$

$$= -\frac{2}{\pi} - 1$$

Most students used the correct expression for average volume. As with the previous part of the question, arithmetic errors, especially when substituting the terminals, caused difficulties. Some students omitted the –1 in the integrand, while others misplaced negative signs or the constant of 2.

Question 7a.

Marks	0	1	2	Average
%	39	15	46	1.1



$$\Pr(F) = \frac{4}{9}\times\frac{1}{20}+\frac{5}{9}\times\frac{2}{25} = \frac{1}{15}$$

While a tree was not a required to answer the question, it may have assisted some students to determine the two required cases. Many students stated probabilities greater than 1.

Question 7b.

Marks	0	1	Average
%	68	32	0.3

$$\Pr(A|F) = \frac{\Pr(A\cap F)}{\Pr(F)} = \frac{\left(\frac{1}{45}\right)}{\left(\frac{1}{15}\right)} = \frac{1}{3}$$

In general the conditional probability was recognised but not the reduced sample space. Often the instruction regarding the form of the final answer was overlooked.

Question 8a.

Marks	0	1	2	Average
%	62	21	17	0.6

$$\frac{d}{dx}\left(\frac{x^k}{k^2}(k\log_e(x)-1)\right)=\left[(k\log_e(x)-1)\times\left(\frac{kx^{k-1}}{k^2}\right)\right]+\left[\frac{x^k}{k^2}\times\frac{k}{x}\right]$$

$$=x^{k-1}\log_e(x)-\frac{x^{k-1}}{k}+\frac{x^{k-1}}{k}$$

$$=x^{k-1}\log_e(x)$$

Hence $\int x^{k-1}\log_e(x)\ dx=\frac{x^k}{k^2}\left(k\log_e(x)-1\right)$

This question was attempted well. Most students applied the product rule but struggled with the algebraic manipulation, often confusing *k* (a constant) with the variable *x*, which gave them an incorrect result. Students who expanded the expression before differentiating or those who made fewer manipulations tended to scored more highly. Some students differentiated the wrong expression.

Question 8bi.

Marks	0	1	2	Average
%	81	7	12	0.3

$$P(X>\frac{1}{e})=\int_{\frac{1}{e}}^{1}-4x\log_e(x)dx$$

$$=-4\left[\frac{x^2}{4}\times(2\log_e(x)-1)\right]_{\frac{1}{e}}^{1}$$

$$=1-\frac{3}{e^2}$$

Students made the connection to part a. and determined *k* = 2. However, few managed to find the correct antiderivative. Evaluation after substituting terminals was problematic. Some used incorrect terminals.

Question 8bii.

Marks	0	1	2	Average
%	87	6	7	0.2

$e > \frac{5}{2}$ so that $\frac{3}{e^2} < \frac{12}{25} < \frac{1}{2}$

So $1 - \frac{3}{e^2} > \frac{1}{2}$

Thus $\Pr\left(X > \frac{1}{e}\right) > \frac{1}{2}$ so that the median is greater than $\frac{1}{e}$

This question was not answered well, despite being well attempted. Often the instruction 'hence' was ignored. Students were required to link to their answer from Question 8bi. by substituting $e > \frac{5}{2}$. Many students based their conclusion on an observation of the graph given in part a., which was not drawn to scale, as per the instructions given at the start of the examination. Some tried, with little success, to evaluate the median, but this was not required. A common misconception was to assume that since $\frac{1}{e} < \frac{2}{5} < \frac{1}{2}$ then $\Pr\left(X > \frac{1}{e}\right) > \frac{1}{2}$.